Determinare l'eq.ne della funzione inversa

$$y = \frac{2x}{x-3}$$

Procedimento:

$$x = \frac{2y}{y-3}$$

$$x(y-3) = 2y$$

$$xy - 3x = 2y$$

$$xy - 2y = 3x$$

$$y(x-2) = 3x$$

$$\boxed{y = \frac{3x}{x-2}}$$

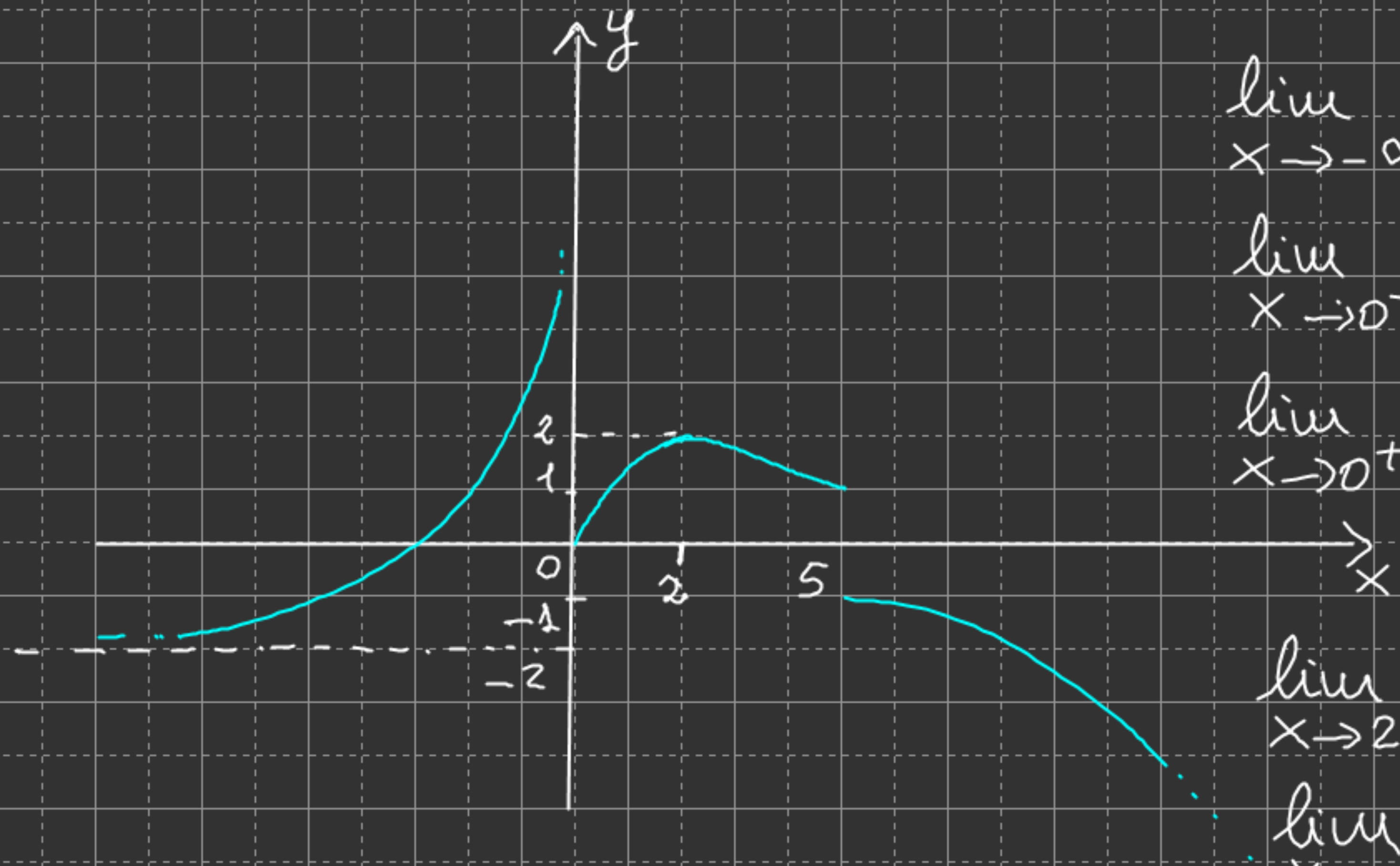
Determinare la funzione composta

$$f(x) = \frac{1}{2}x + 3 \qquad g(x) = x^2$$

$$(f \circ g)(x) = f(g(x)) = f(x^2) = \frac{1}{2}x^2 + 3$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{2}x + 3\right) = \left(\frac{1}{2}x + 3\right)^2 =$$

$$= \frac{1}{4}x^2 + 3x + 9$$



$$\lim_{x \to -\infty} f(x) = -2$$

$$\lim_{x \to 0^-} f(x) = +\infty$$

$$\lim_{x \to 0^+} f(x) = 0$$

$$\lim_{x \to 2} f(x) = 2$$

$$\lim_{x \to 5^-} f(x) = 1$$

$$\lim_{x \to 5^+} f(x) = -1 \qquad \lim_{x \to 5} f(x) = \nexists$$

Da soli:

1) Fne inversa di $y = \frac{4-x}{5x}$

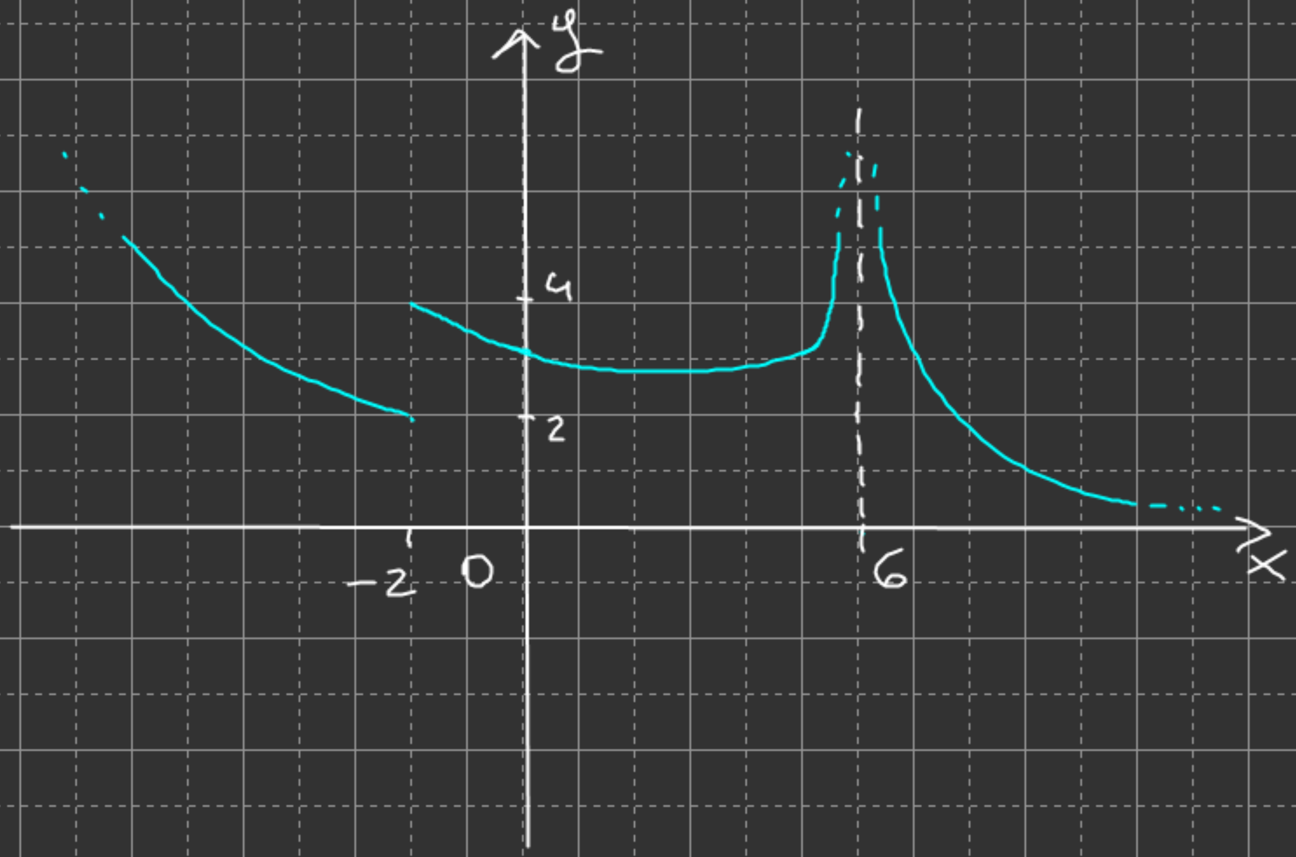
2) Det. $f \circ g$ e $g \circ f$, quando $f(x) = x^2+1$ e $g(x) = \sqrt{x}$

3) Studio completo di

a) $y = \frac{x-3}{x^4-3x^2+2}$

b) $y = \sqrt{\frac{x^5}{x^2+2x}}$

(Approfondimento)



$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to -2^-} f(x) = 2$$

$$\lim_{x \to -2^+} f(x) = 4$$

$$\lim_{x \to -2} f(x) = \nexists$$

$$\lim_{x \to 6^-} f(x) = +\infty$$

$$\lim_{x \to 6^+} f(x) = +\infty$$

$$\lim_{x \to 6} f(x) = +\infty \quad \lim_{x \to +\infty} f(x) = 0$$

1) $y = \frac{4-x}{5x}$

$x = \frac{4-y}{5y}$

$5xy = 4 - y$

$y(5x+1) = 4$

$y = \frac{4}{5x+1}$

2) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 =$

$= x + 1$

$(g \circ f)(x) = g(f(x)) = g(x^2+1) =$

$= \sqrt{x^2+1}$

3a) $y = \dfrac{x-3}{x^4-3x^2+2}$

- Fne alg raz. fratta

- Dominio

$x^4-3x^2+2=0$

$x^2=t$

$t^2-3t+2=0$

$t = \dfrac{3\pm\sqrt{9-8}}{2} = \dfrac{3\pm 1}{2}$

$1 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

$2 \Rightarrow x^2=2 \Rightarrow x=\pm\sqrt{2}$

$\Rightarrow D = \mathbb{R}-\{\pm 1, \pm\sqrt{2}\}$

• Simmetrie

$$f(-x) = \frac{-x-3}{(-x)^4-3(-x)^2+2} = \frac{-x-3}{x^4-3x^2+2} \neq \pm f(x)$$ né p. né d.

• Int. assi

asse x: $x-3=0 \Rightarrow x=3 \Rightarrow (3,0)$

asse y: $y = -\frac{3}{2} \Rightarrow \left(0,-\frac{3}{2}\right)$

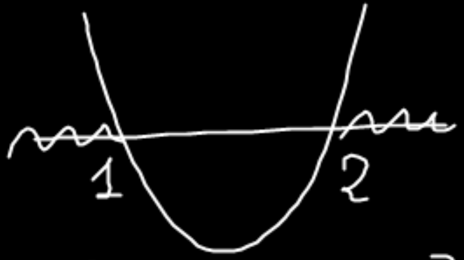
- Segno

$$\frac{x-3}{x^4-3x^2+2}>0$$

N. $x-3>0$ $x>3$

D. $x^4-3x^2+2>0$

$t^2-3t+2>0$

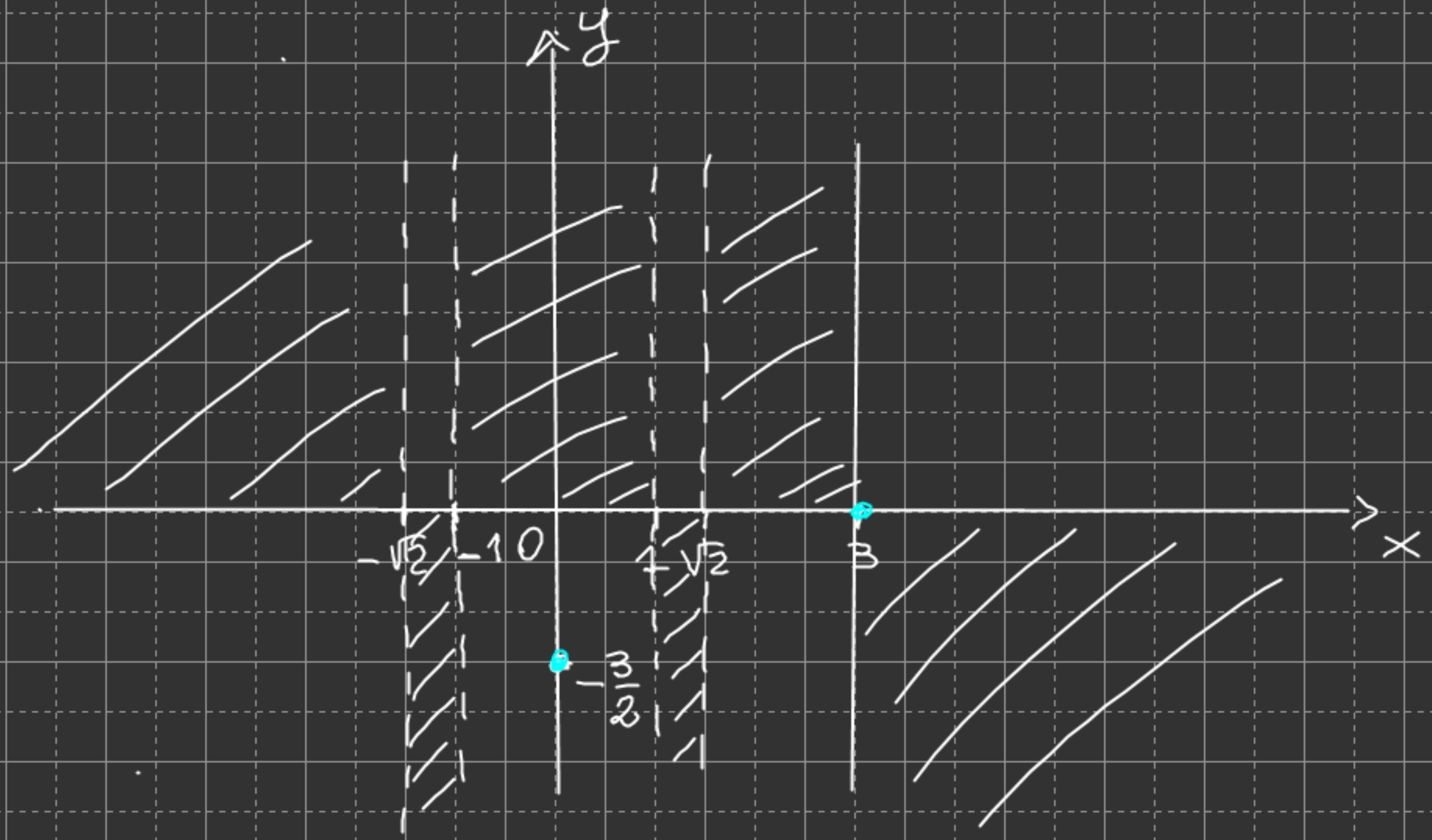


$x^2<1 \lor x^2>2$

$-1<x<1 \lor x<-\sqrt{2} \lor x>\sqrt{2}$

TABELLA

		$-\sqrt{2}$		-1		1		$\sqrt{2}$		3	
N	−		−		−		−		−	0	+
D	+	0	−	0	+	0	−	0	+		+
$\frac{N}{D}$	−	∄	+	∄	−	∄	+	∄	−	0	+



y
x
$-\sqrt{2}$
-1
0
1
$\sqrt{2}$
3
$-\frac{3}{2}$