

Studio di funzione:

classificazione, dominio, simmetrie, int. assi,

segno, grafico

$$y = x^3 - 9x$$

Fme algebrica razionale intera

$$D = \mathbb{R}$$

$$f(-x) = (-x)^3 - 9(-x) = -x^3 + 9x = -f(x) \Rightarrow \text{DISPARI}$$

(e' simmetrica rispetto all'origine)

$$\text{Int. asse } x: x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x = 0$$

$$x^2 - 9 = 0$$

$$x = \pm 3$$

$$\Rightarrow (0,0) \quad (-3,0)$$

$$(3,0)$$

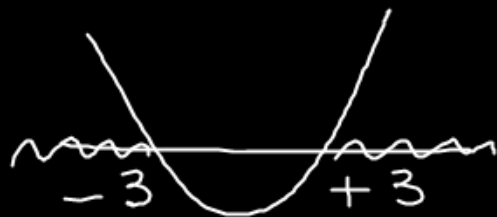
• Segno

$$x^3 - 9x > 0$$

$$x(x^2 - 9) > 0$$

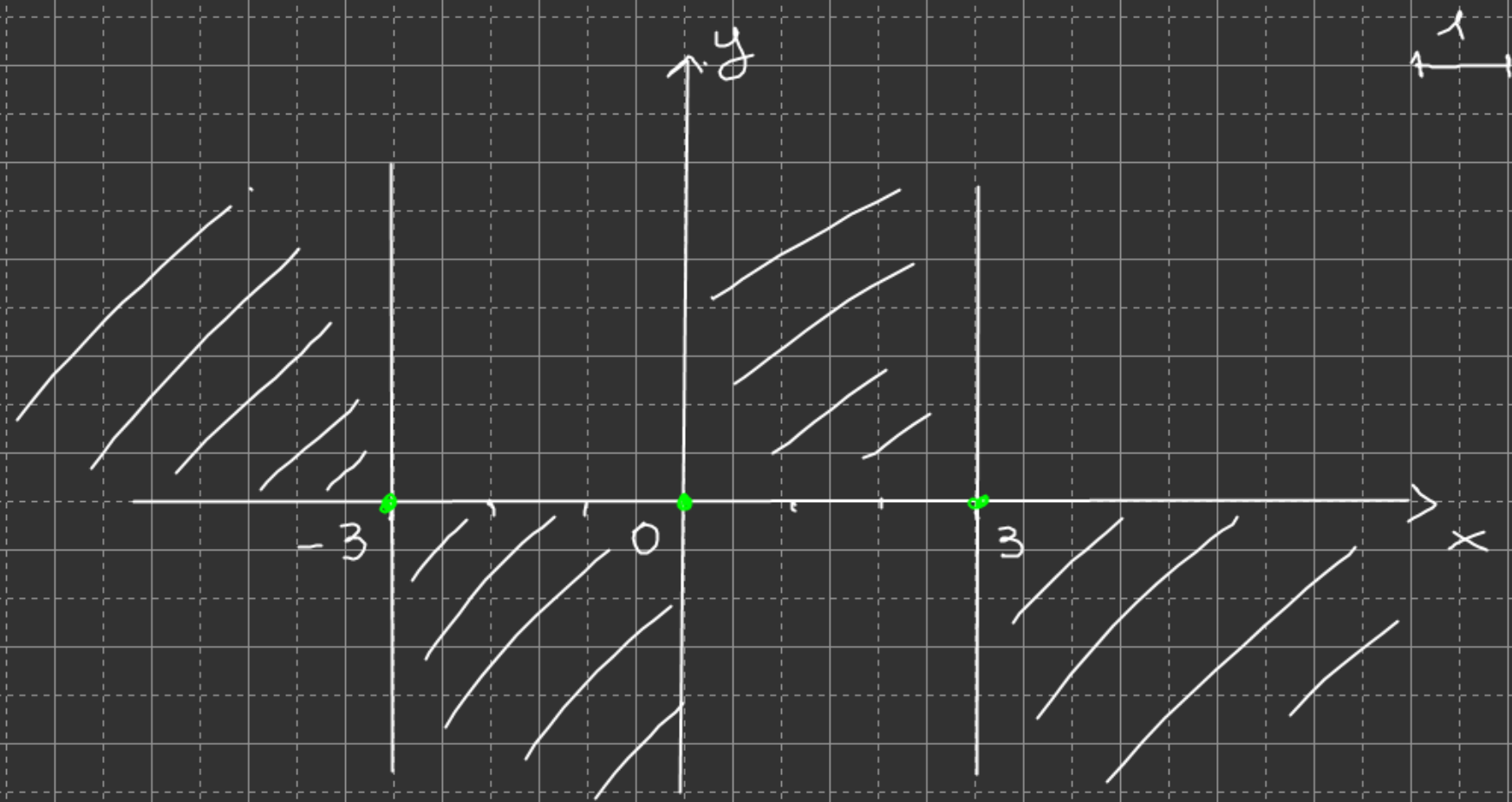
•  $x > 0$

•  $x^2 - 9 > 0$



$$x < -3 \vee x > 3$$

	-3	0	3	
F1	-	-	+	+
F2	+	-	-	+
P	-	+	-	+



$$y = \frac{1-x}{x^2-2x}$$

• Fme algebrica razionale fatta

• Dominio

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0$$

$$x - 2 = 0 \quad x = 2$$

$$\Rightarrow D = \mathbb{R} - \{0, 2\}$$

• Simmetrie

$$f(-x) = \frac{1 - (-x)}{(-x)^2 - 2 \cdot (-x)} = \frac{1+x}{x^2+2x} \neq \pm f(x) \Rightarrow \text{né pari} \\ \text{né dispari}$$

• Int. axe x :  $\frac{1-x}{x^2-2x} = 0 \Rightarrow 1-x=0 \Rightarrow x=1$   
 $(1,0)$

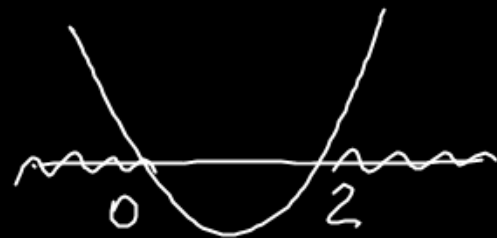
Segno

$$\frac{1-x}{x^2-2x} > 0$$

N.  $1-x > 0$

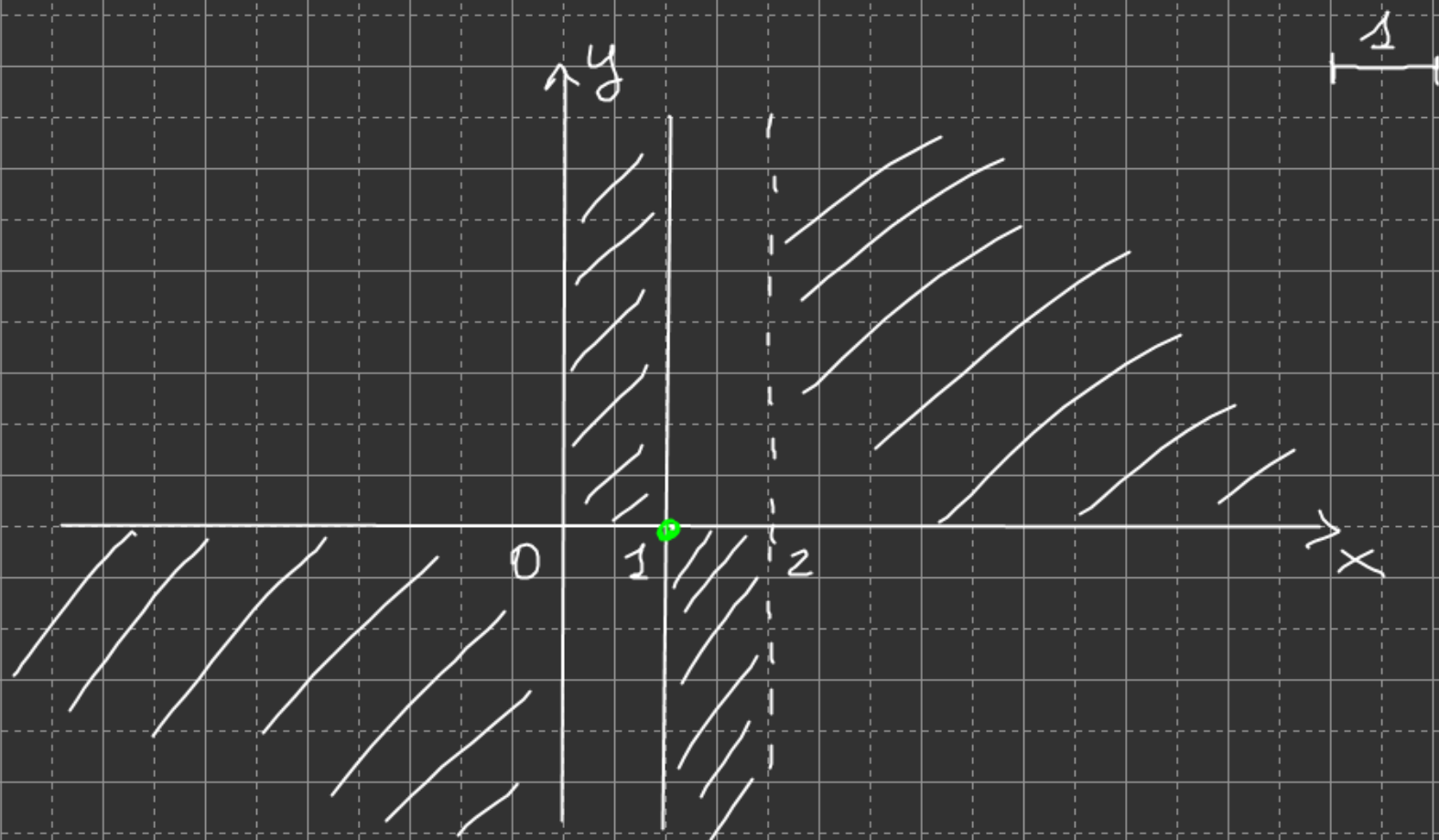
$$x < 1$$

D.  $x^2-2x > 0$



$$x < 0 \vee x > 2$$

	0	1	2	
N	+	+	-	-
D	+	-	-	+
$\frac{N}{D}$	<del>+</del>	-	<del>+</del>	-





$$1) y = x^4 - 8x$$

$$2) y = \frac{x^2 - 5x + 4}{x + 2}$$

Studio (1)

• Fme alg. raz. intera •  $D = \mathbb{R}$

•  $f(-x) = (-x)^4 - 8(-x) = x^4 + 8x \neq \pm f(x)$  né pari né disp.

• Int. axe x:  $x^4 - 8x = 0$   
 $x(x^3 - 8) = 0$

$$x = 0$$

$$\Rightarrow (0, 0)$$

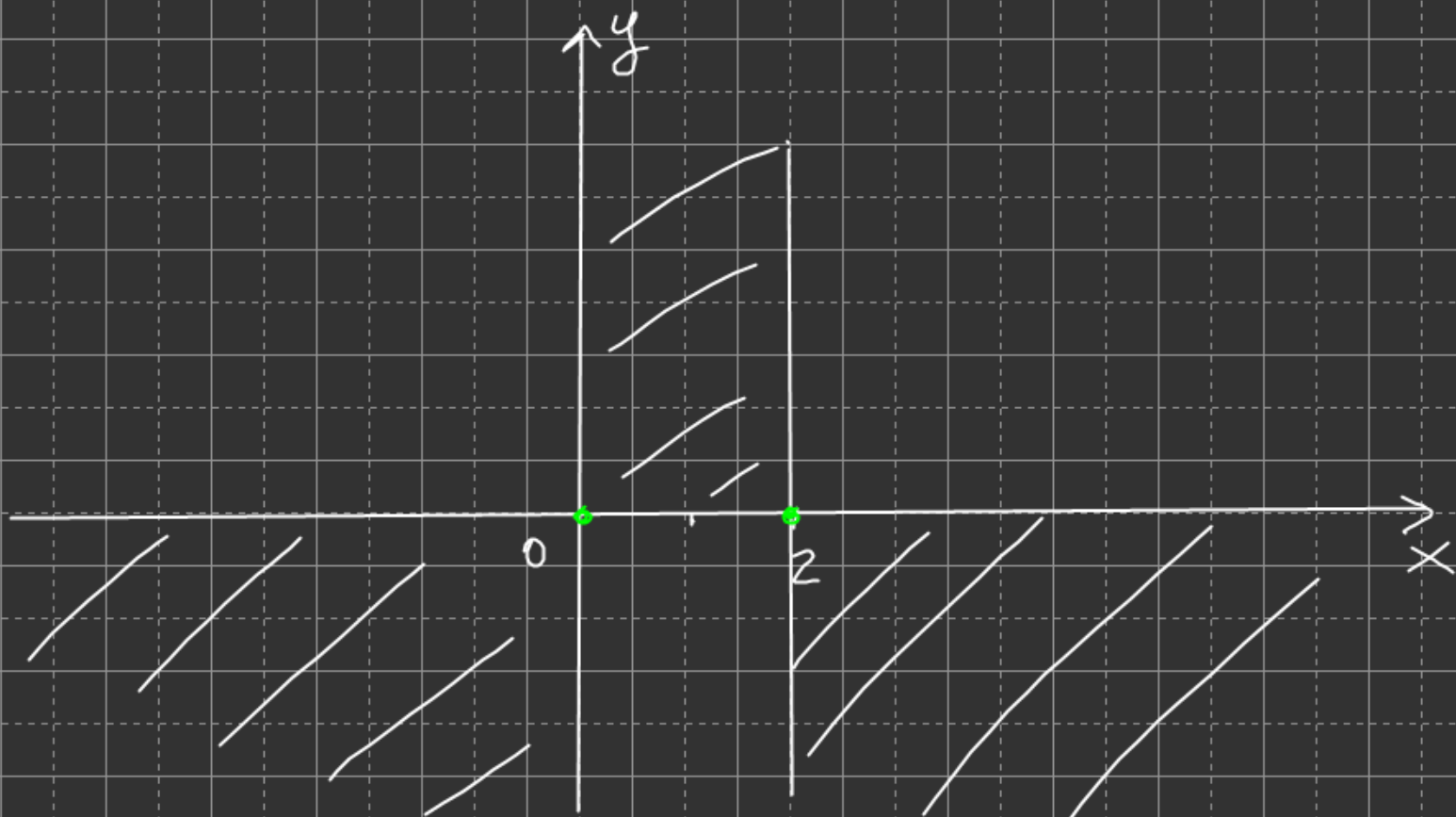
$$x^3 - 8 = 0$$

$$x^3 = 8$$

$$x = 2$$

$$(2, 0)$$





Studio  $y = \frac{x^2 - 5x + 4}{x + 2}$

• Fine alg. ras. fatta •  $D = \mathbb{R} - \{-2\}$

•  $f(-x) = \frac{x^2 + 5x + 4}{-x + 2} \neq \pm f(x) \Rightarrow$  né pari né dispari

• Int. asse  $x$ :  $\frac{x^2 - 5x + 4}{x + 2} = 0 \Rightarrow x^2 - 5x + 4 = 0$

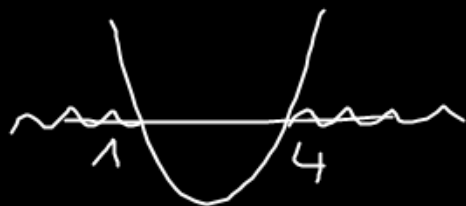
$$x = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} \begin{cases} 1 \\ 4 \end{cases} \Rightarrow \begin{matrix} (1, 0) \\ (4, 0) \end{matrix}$$

• Int. asse  $y$ :  
(sostituisco 0 alla  $x$ )  $y = \frac{4}{2} = 2 \Rightarrow (0, 2)$

• Segno

$$\frac{x^2 - 5x + 4}{x + 2} > 0$$

N.  $x^2 - 5x + 4 > 0$

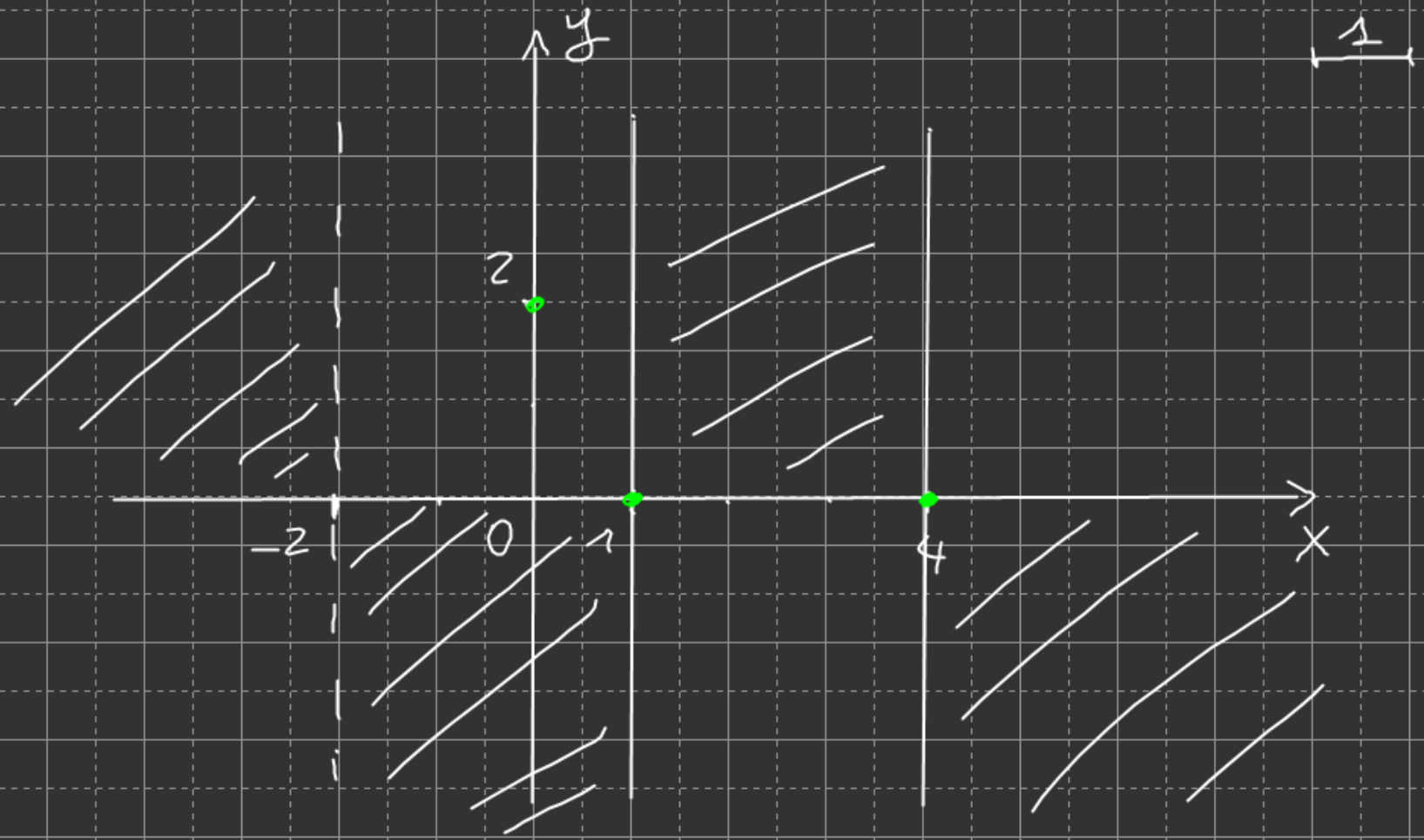


$$x < 1 \vee x > 4$$

D.  $x + 2 > 0$

$$x > -2$$

	-2		1		4		
N	+		+	o	-	o	+
D	-	o	+		+		+
<del>D</del>	-	<del>o</del>	+	o	-	o	+



Domínio di funzioni irrazionali:

$$a) y = \sqrt{\frac{2x-6}{x}}$$

$$\frac{2x-6}{x} \geq 0$$

$$N \quad 2x-6 > 0 \quad x > 3$$

$$D \quad x > 0$$

N	-	0	-	3	+
D	-	0	+	0	+
N/D	+	<del>0</del>	-	0	+

$$D = \{ x < 0 \vee x \geq 3 \}$$

$$b) y = \frac{\sqrt{x+7}}{2x}$$

$$\begin{cases} x+7 \geq 0 \\ x \neq \frac{0}{2} \end{cases} \quad \begin{cases} x \geq -7 \\ x \neq 0 \end{cases}$$

$$D = \{x \geq -7 \wedge x \neq 0\}$$

$$c) y = \frac{3x+9}{\sqrt{x}}$$

$$\begin{cases} x \geq 0 \\ x \neq 0 \end{cases} \quad x > 0$$

$$D = \{x > 0\} = \mathbb{R}^+$$



$$d) y = \sqrt[3]{\frac{2x-6}{x}}$$

Se l'indice è dispari, studio il dominio del

$$\text{radicando : } y = \frac{2x-6}{x} \Rightarrow D = \mathbb{R} - \{0\}$$

$$e) y = \sqrt{4-x} + \sqrt{x}$$

$$\begin{cases} 4-x \geq 0 \\ x \geq 0 \end{cases} \quad \begin{cases} x \leq 4 \\ x \geq 0 \end{cases}$$

$$D = \{0 \leq x \leq 4\}$$

Da soli:

$$y = \sqrt{x^2 + 4} + \frac{1}{x+3} ; y = \sqrt[3]{x^2 - 6x} ;$$

$$y = \sqrt{\frac{2x^2 + x - 1}{x - 1}} ; y = \frac{\sqrt{2x^2 + x - 1}}{x - 1} ;$$

$$y = \frac{\sqrt[3]{2x^2 + x - 1}}{\sqrt{x - 1}}$$

$$\boxed{1} \begin{cases} x^2 + 4 \geq 0 \\ x + 3 \neq 0 \end{cases} \begin{cases} \mathbb{R} \\ x \neq -3 \end{cases}$$

$$D = \mathbb{R} - \{-3\}$$

$$\boxed{2} D = \mathbb{R}$$

$$\boxed{3} \frac{2x^2 + x - 1}{x - 1} \geq 0$$

	-	1		1/2		1	
N	+	0	-	0	+	0	+
D	-	0	-	0	-	0	+
N/D	-	0	+	0	-	0	+

$$N. \quad 2x^2 + x - 1 > 0$$

$$x < -1 \quad \vee \quad x > \frac{1}{2}$$

$$D. \quad x - 1 > 0$$

$$x > 1$$

$$D = \left\{ -1 \leq x \leq \frac{1}{2} \quad \vee \quad x > 1 \right\}$$

$$\boxed{14} \quad \begin{cases} 2x^2 + x - 1 \geq 0 \\ x - 1 \neq 0 \end{cases} \quad \begin{cases} x \leq -1 \vee x \geq \frac{1}{2} \\ x \neq 1 \end{cases}$$
$$D = \left\{ x \leq -1 \vee x \geq \frac{1}{2} \wedge x \neq 1 \right\}$$

$$\boxed{15} \quad x - 1 > 0 \quad D = \{x > 1\}$$

$$\hookrightarrow y = \frac{\sqrt[3]{2x^2 + x - 1}}{\sqrt{x - 1}}$$

Al più veloci:

dominio di: ①  $y = \frac{1}{1 - \ln x}$  ; ②  $y = \frac{\ln x}{\sqrt{x^2 - 25}}$  ;

③  $y = 3^{\sqrt{x^2 - 4}} + \frac{1}{6 + x}$  ;

④  $y = \sqrt{\ln(x + 3)}$

$$\textcircled{1} \begin{cases} x > 0 \\ \ln x \neq 1 \end{cases} \quad \begin{cases} x > 0 \\ x \neq e \end{cases} \quad D = \{x > 0 \wedge x \neq e\} = \mathbb{R}^+ - \{e\}$$

$$\textcircled{2} \begin{cases} x > 0 \\ x^2 - 25 > 0 \end{cases} \quad \begin{cases} x > 0 \\ x < -5 \vee x > 5 \end{cases} \quad D = \{x > 5\}$$

$$\textcircled{3} \begin{cases} x^2 - 4 \geq 0 \\ 6 + x \neq 0 \end{cases} \quad \begin{cases} x \leq -2 \vee x \geq 2 \\ x \neq -6 \end{cases} \quad D = \{x \leq -2 \vee x \geq 2 \wedge x \neq -6\}$$

$$\textcircled{4} \ln(x+3) \geq 0 \quad \Rightarrow x+3 \geq 1 \quad \Rightarrow x \geq -2 \quad D = \{x \geq -2\}$$